LESSON PLAN

Solar Power and Me: The Inherent Advantages

Grade Level:
8-12

Subjects:
Math: Algebra I and II

Length:
90 Minutes
Solar Power and Me:
The Inherent Advantages

INTRODUCTION

This is a uniquely interdisciplinary high school algebra and solar energy lesson geared toward an Algebra I-II class. It uses data from a 2.1-kilowatt photovoltaic solar panel system at a high school in rural North Carolina, historical energy statistics from the U.S. Energy Information Administrations (EIA) on solar and renewable energy growth in the U.S., and the financial savings accrued from a residential solar photovoltaic system to teach students the basics of renewable energy and best-fit regression mathematical models.

In this lesson, students will learn how photovoltaic solar panel systems work, and create best-fit mathematical regression models using a TI-83 or TI-84 calculator to make predictions and solve real-life problems concerning solar energy in the United States.

This lesson assumes little to no prior knowledge of photovoltaic solar power systems or renewable energy, thus it has the potential to benefit all levels of Algebra students regardless of their scientific background. The lesson includes two pages of background reading with four questions on solar power that can be used as primer for students who are not knowledgeable about solar energy.

LESSON OVERVIEW

Grade Level & Subject: Grades 8-12, Math: Algebra I and Algebra II

Length: 90 minutes; 1-3 class periods, depending on length and ability level of class

Objectives:

After completing this lesson, students will be able to:

- Create and use best-fit mathematical models of quadratic functions to solve problems involving sets of data.
- Identify and recognize the key concepts of photovoltaic solar power as a renewable energy source.
- Understand the relationship between mathematics and solar power.
National Standards Addressed:

This lesson addresses the following National Science Education Standards from the National Academies of Science:

- **Content Standard: NS.5-8.6 PERSONAL AND SOCIAL PERSPECTIVES**
  In grades 5-8, all students should develop an understanding of:
  1. Populations, resources, and environments
  2. Risks and benefits
  3. Science and technology in society

This lesson addresses the following National Education Standards from the National Council of Teachers of Mathematics:

- **Content Standard 1: Mathematics as Problem Solving**
  In grades 8-11, the mathematics curriculum should include the refinement and extension of methods of mathematical problem-solving so that all students can:
  - Use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content
  - Apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics
  - Apply the process of mathematical modeling to real-world situations

- **Content Standard 5: Algebra**
  In grades 8–11, the mathematics curriculum should include the continued study of algebraic concepts and methods so that all students can:
  - Represent situations that involve variable quantities with expressions, equations, inequalities, and matrices
  - Use tables and graphs as tools to interpret expressions, equations, and inequalities
  - Demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations

- **Content Standard 6: Functions**
  In grades 8–11, the mathematics curriculum should include the continued study of functions so that all students can:
  - Model real-world phenomena with a variety of functions
  - Recognize that a variety of problems can be modeled by the same type of function.

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1 National Science Education standards are from the National Academies of Science, 2011: http://www.nas.edu. National Science Education standards can also be found at: http://www.education-world.com/standards.
21st Century Skills:

This lesson addresses the following 21st Century Skills from the Partnership for 21st Century Skills by asking students to:

- Focus on 21st century skills, content knowledge, and expertise.
- Engage with real-world data, tools, and experts they will encounter in college, on the job, and in life. Students learn best when actively engaged in solving meaningful problems.
- Investigate and analyze environmental issues, and make accurate conclusions about effective solutions.
- Demonstrate knowledge and understanding of society’s impact on the natural world (e.g., population growth, population development, resource consumption rate, etc.).
- Develop a deep understanding rather than shallow knowledge.

Key Questions:

- Can students calculate the regression model?
- Can students understand the meaning of the coefficients within the model?
- Can students state the model’s goodness of fit?
- Can students use the model to make useful predictions?

Materials Needed:

- TI-83 or 84 graphing calculator
- Computer with Internet access
- Printer
- Reproducible #1 – Warm-Up – Energy for Life
- Reproducible #2 – Warm-Up – Energy for Life – Answer Key
- Reproducible #3 – All about Solar! – Background Info and Questions
- Reproducible #4 – All about Solar! – Answer Key
- Reproducible #5 – Quadratic Graphs and Solar Savings
- Reproducible #6 – Quadratic Graphs and Solar Savings – Answer Key
- Reproducible #7 – Exponential Models: Solar PV System
- Reproducible #8 – Exponential Models: Solar PV System – Answer Key
- Reproducible #9 – Best-Fit Models: U.S. Energy Consumption
- Reproducible #10 – Best-Fit Models: U.S. Energy Consumption – Answer Key
- Reproducible #11 – Solar Power and Me: The Financial Savings
- Reproducible #12 – Solar Power and Me: The Financial Savings – Answer Key
- Reproducible #13 – Exit Ticket Question* (Note: this reproducible has two copies on a single page, so please cut in half before distributing to students.)
- Reproducible #14 – Exit Ticket Question – Answer Key
- Reproducible #15 – Appendix A– Calculator Regression Model Guide for Students

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Assessment:

Students will be assessed through the following activities:

- Completion of class work assignments:
  - Reproducible #1 – Warm-Up Energy for Life
  - Reproducible #3 – All about Solar – Background Info and Questions
  - Reproducible #5 – Quadratic Graphs and Solar Savings
  - Reproducible #7 – Exponential Models: Solar PV System
  - Reproducible #9 – Best-Fit Models: U.S. Energy Consumption
  - Reproducible #11 – Solar Power and Me: The Financial Savings
  - Reproducible #13 – Exit Ticket Question
- Completion of Exit Ticket (5 questions/5 minutes)
- Completion of extension work (homework or extra credit)

LESSON BACKGROUND

Relevant Vocabulary:

- **Coefficient**: A number or quantity placed (generally) before another multiplying quantity or variable in an algebraic term, such as the number 3 in the expression 3x.\(^4\)
- **Correlation**: A number or function that describes the degree of relationship between two sets of data or two random variables.\(^5\)
- **Dependent Variable**: A mathematical variable whose value is determined by the value of an independent variable.\(^6\)
- **Independent Variable**: A mathematical variable that is not dependent on other variables.\(^7\)
- **Nonrenewable Energy**: A resource that cannot be produced, re-grown, regenerated, or reused as fast as it is consumed. These resources exist in fixed amounts and are used up faster than they can be replaced in nature.\(^8\)
- **Photovoltaic Effect**: The phenomenon in which the incidence of light or other electromagnetic radiation upon the junction of two dissimilar materials, as a metal and a semiconductor the generation of an electromotive force (electricity!). In essence, it is the generation of electricity through the exposure of material to light. Photo = light, voltaic =

electricity.\(^9\)

- **Photovoltaic Cell (Solar Cell):** A thin semiconductor wafer specially treated to form an electric field, positive on one side and negative on the other. When light energy strikes the PV cell, electrons are knocked loose from the atoms in the semiconductor material. If electrical conductors are attached to the positive and negative sides, forming an electrical circuit, the electrons can be captured in the form of an electric current (electricity). This electricity can then be used to power a load, such as a light or a tool.\(^{10}\)

- **R\(^2\):** The coefficient of determination, \(r^2\), gives the proportion of the variance (fluctuation) of one variable that is predictable from another variable. This measurement allows us to determine how certain we can be in making predictions from certain models or graphs.\(^{11}\)

- **Regression:** The process of obtaining the line of best fit. The relationship between two sets of data may be described with a line using goodness-of-fit criterion. The regression line may be used to describe data and make predictions.\(^{12}\)

- **Renewable Energy:** Any naturally occurring, theoretically inexhaustible source of energy, such as biomass, solar, wind, tidal, wave, and hydroelectric power, that is not derived from fossil or nuclear fuel.\(^{13}\)

- **Residuals:** The difference between a value measured in a scientific experiment and the theoretical or true value.\(^{14}\)

- **Scatter Plot:** A graph in which the values of two variables are plotted along two axes. The pattern of the resulting points reveals any correlation present.\(^{15}\)

- **Solar Energy:** Solar energy is the solar radiation from the sun’s rays that reach the earth. This energy can be converted into other forms of energy, such as heat and electricity. Passive solar energy can be exploited by positioning a house to allow sunlight to enter through the windows to help heat a space. Active solar energy involves the conversion of sunlight to electrical energy, especially in solar (photovoltaic) cells.\(^{16}\)

- **Solar Panel:** A panel designed to absorb the sun’s ray as a source of energy for generating electricity or heat.\(^{17}\)

- **Watt:** A basic unit of power, equivalent to one joule per second, corresponding to the power in an electric circuit in which the potential difference is one volt and the current one

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\(^15\) *Scatter Plot Entry*. Oxford University Press Online Dictionary. Retrieved May 18\(^{th}\), 2011 from http://oxforddictionaries.com/search?searchType=dictionary&isWritersAndEditors=true&searchUri=All&q=scatter+plot&searchBn=Search&contentVersion=US.


ampere.18

Background Information:

Please see Reproducible #2 – All about Solar! for background information on solar energy.

Resources:

- **Solar Decathlon Educational Resources for Teachers** – U.S. Department of Energy
- **Solar and Photovoltaic cells** – U.S. Department of Energy
- **Solar America Cities** – U.S. Department of Energy
- **Solar Basics Energy for Kids** – U.S. Energy Information Administration
- **Solar Explained** – U.S. Energy Information Administration
- **Solar Energy Basics** – National Renewable Energy Laboratories

**LESSON STEPS**

Warm-up: *Earth’s Limited Resources*

1. Begin this lesson by handing out Reproducible #1 – Energy for Life and reviewing the instructions with students. Tell students that today’s lesson will apply mathematical skills to new ideas and technologies concerning the environment.

2. Ask students to work in pairs to complete the answers. Allow 10 – 15 minutes for this exercise.

3. Begin a short discussion based on the questions on the worksheet.
   - How do our individual choices affect the earth’s natural resources?
     *Answers will vary, but may include driving using more efficient cars, saving electricity/water, and using only the resources you need.*
   - How can technology help or hinder this?
     *Energy-efficient and renewable energy technologies can be used to reduce our reliance on natural resources.*
   - What are some ways each individual can reduce his or her impact?
     *Answers will vary, but may include recycling, turning off the lights and turning off electronics when not in use, using public transportation, choosing energy-efficient products, and using renewable energy.*
   - What are some ways the broader society can reduce its impact?
     *Answers will vary, but may include decreasing reliance on non-renewable energy sources and switching to renewable ones.*

Activity One: What is Solar Energy?

1. Begin this activity by asking students a few questions:
   - What is renewable energy?
     Renewable energy is any naturally occurring, theoretically inexhaustible source of energy, such as biomass, solar, wind, tidal, wave, and hydroelectric power that is not derived from fossil or nuclear fuel.
   - Which of the sources of energy mentioned in the warm-up (hydroelectric, coal, geothermal, wind, solar, biomass, natural gas, petroleum/oil and nuclear) are renewable?
     Hydroelectricity, geothermal, wind, solar, and biomass.
2. Explain to students that today’s lesson will focus on solar energy.
   Solar energy is the sun’s rays (solar radiation) that reach the earth. This energy can be converted into other forms of energy, such as heat and electricity. Passive solar energy can be exploited by positioning a house to allow sunlight to enter through the windows to help heat a space. Active solar energy involves the conversion of sunlight to electrical energy, especially in solar (photovoltaic) cells.
3. Hand out Reproducible #2 – All About Energy – Background Info and Questions.
4. Read the background information together as a class. Allow time for students to review and ask questions.
5. Ask students to complete the worksheet by answering the questions at the bottom. Allow 10-15 minutes to complete the worksheet.
6. Ask students why they think solar energy use is not more common. What can be done to encourage solar energy use? See answers below on Reproducible #3 – All About Solar – Answer Key.

Activity Two: Modeling Solar Energy

1. Tell students that they’re now going to analyze an average homeowner’s energy use to understand the economic benefits of solar energy.
2. Hand out Reproducible #3 – Best-Fit Models: Solar Savings. Display either Appendix A or Appendix B on the overhead projector, Smart Board, or equivalent or pass out copies so students can follow along while making their calculations.
3. Ask students to use their TI-83 or 84 graphing calculators to complete the worksheet.
4. After allowing students 20 – 25 minutes to finish their calculations, review the answers. Make sure students understand how their figures translate into savings.

Activity Three: Solar Energy in the U.S.

1. Explain to students that since they now know about energy and dollar savings on an individual home, it’s time to look at the energy consumption of the U.S. as a whole.
2. Hand out Reproducible #4 – Lines of Best Fit: U.S. Energy Consumption and Reproducible #8 – Appendix A and Appendix B.
3. Ask students to complete Reproducible #4 by using their TI-83 or TI-84 calculators and the appendices as a guide. If necessary, have students follow along on their own calculators as you read through Appendix A and Appendix B and answer the questions together. Steps and answers can be found on Reproducible #9 – Appendix C – Guided Calculator Notes for Teachers with Answer Key. Then have students complete Reproducible #4 on their own.
4. Review answers with the class.
Wrap Up: *Solar Energy in the Real World*

1. Hand out **Reproducible #5 – Exit Ticket Question** to students.
2. Have students fill in the answers using the skills learned in the previous activities.
3. Engage students in a discussion about solar energy. Ask the following questions:
   - What have you learned about solar energy?
     *Answers will vary.*
   - Should governments invest in solar energy? Why or why not?
     *Answers will vary.*
   - How can we best translate the mathematical models from the lesson into a potential real-life situation to help explain the potential benefits of solar energy?
     *Answers will vary, but may include how proper mathematical models could increase the efficiency of a specific photovoltaic solar panel system.*

Extension: *Solar Cities*

1. Ask students to review the U.S. Department of Energy’s *Solar America Communities* website either for homework the night before or during class if there is computer access.\(^{19}\) Tell them to take note of how solar energy is incorporated into the infrastructure of various cities nationwide and write a short essay about how they would encourage solar energy use in their own town.
2. Have students research solar energy use by other countries and regions. Use a best-fit equation to predict the consumption of solar energy in Europe in 2013. What about China? Start a discussion about why it would be important to know how much solar energy other countries use.

**CONCLUSION**

Students should leave this lesson better able to use TI-83 or TI-84 calculators to produce and compare polynomial regression equations to model real-life situations, and able to make mathematically relevant predictions. Students should also understand the environmental and economic benefits of renewable resources, and how solar energy is captured and used in a photovoltaic system.

**LESSON PLAN CREDITS**

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REPRODUCIBLES FOR CLASSROOM USE

This lesson plan contains the following reproducible documents for classroom use:

- Reproducible #1 – Warm-Up – Energy for Life
- Reproducible #2 – Warm-Up – Energy for Life – Answer Key
- Reproducible #3 – All about Solar! – Background Info and Questions
- Reproducible #4 – All about Solar! – Answer Key
- Reproducible #5 – Quadratic Graphs and Solar Savings
- Reproducible #6 – Quadratic Graphs and Solar Savings – Answer Key
- Reproducible #7 – Exponential Models: Solar PV System
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- Reproducible #13 – Exit Ticket Question* (Note: this reproducible has two copies on a single page, so please cut in half before distributing to students.)
- Reproducible #14 – Exit Ticket Question – Answer Key
- Reproducible #15 – Appendix A– Calculator Regression Model Guide for Students
- Reproducible #16 – Appendix B – Extension Learning Opportunities for Students
- Reproducible #17 – Appendix C – Guided Calculator Notes for Teachers with Answer Key
Warm Up – Energy for Life

Essential Question: How difficult is it to predict our energy future?

Directions: Thoroughly read the information below, and then answer the questions to the best of your ability in at least two complete sentences.

The earth has limited natural resources to support life. An increasing human population coupled with a dramatic increase in the quality of life for many people worldwide is increasing pressure on these resources. Economic growth requires energy, but our ever-increasing demand for energy is depleting resources and damaging ecosystems worldwide. The Deepwater Horizon oil spill in the Gulf of Mexico in 2010 is a powerful example.

The choices individuals, businesses, governments, and organizations make can protect natural resources and improve our environment. However, sometimes those choices aren’t easy and they often require an assessment of the costs and benefits.

1. How do you think your daily actions affect the earth’s natural resources?

2. How can technology help or hinder our ability to maintain a healthy and safe planet while supporting economic growth?

3. What percent of our energy do you think comes from coal, and what percent do you think comes from solar panel systems, such as the ones you see on houses and schools? Do you think these percentages are changing?

4. Make a prediction: what do you think the planet will look like when you are 50 years old?
5. Make a prediction: If you can generate 200 kilowatts of power from a solar panel system in July, 100 kilowatts in December, and 115 kilowatts in January, how many kilowatts do you think the system will produce in February? How did you make such a prediction? Explain in at least two complete sentences.

________________________________________________________________________
________________________________________________________________________

6. The primary energy sources today are coal, renewables (such as solar, wind, hydropower), natural gas, petroleum (oil), and nuclear. Make an educated guess about how much of each type of energy the world uses, and list them below in order of use, from least to greatest. Write the percent in the second column.

<table>
<thead>
<tr>
<th>Type</th>
<th>Amount (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.___________________</td>
<td>__________</td>
</tr>
<tr>
<td>2.___________________</td>
<td>__________</td>
</tr>
<tr>
<td>3.___________________</td>
<td>__________</td>
</tr>
<tr>
<td>4.___________________</td>
<td>__________</td>
</tr>
<tr>
<td>5.___________________</td>
<td>__________</td>
</tr>
</tbody>
</table>
Warm Up – Energy for Life

Answer Key

1. How do you think your daily actions affect the earth’s natural resources?
   Answers will vary.

2. How can technology help or hinder our ability to maintain a healthy and safe planet while supporting economic growth?
   Answers will vary.

3. What percent of our energy use do you think comes from coal, and what percent do you think comes from solar panel systems, such as the ones you see on houses and schools? Do you think these percentages are changing?
   Answers will vary.

4. Make a prediction: what do you think the planet will look like when you are 50?
   Answers will vary.

5. Make a prediction: If you can generate 200 kilowatts of power from a solar panel system in July, 100 kilowatts in December, and 115 kilowatts in January, how many kilowatts do you think the system will produce in February? How did you make such a prediction? Explain in at least two complete sentences.
   144.7 kW.

6. The primary energy sources today are coal, renewables (such as solar, wind, hydropower), natural gas, petroleum (oil), and nuclear. Make an educated guess about how much of each type of energy the world uses, and list them below in order of use, from least to greatest. Write the percent in the second column.

<table>
<thead>
<tr>
<th>Type</th>
<th>Amount (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Petroleum (Oil)</td>
<td>37%</td>
</tr>
<tr>
<td>2. Coal</td>
<td>21%</td>
</tr>
<tr>
<td>3. Natural Gas</td>
<td>25%</td>
</tr>
<tr>
<td>4. Nuclear</td>
<td>9%</td>
</tr>
<tr>
<td>5. Renewables</td>
<td>8%</td>
</tr>
</tbody>
</table>

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All About Solar!

What's the big deal with solar energy?
Over 85 percent of the energy consumed around the world today originates from fossil fuels such as oil, coal, and natural gas. Many prominent scientists predict that if we continue using fossil fuels at the rate we do today, we will considerably deplete such resources in the next 40 – 70 years. So what can we do? Though many options exist, solar power is one of the best.

What are the benefits?
The sun is incredibly powerful. The amount of solar energy reaching the surface of the planet in one year is about twice as much as we will ever obtain from all non-renewable resources (such as coal, oil and natural gas) combined.

Also, unlike fossil fuels, solar energy is a renewable resource. One of the biggest advantages of solar energy is that it is plentiful and essentially free. That means we'll never run out!

Solar power is also much cleaner than power produced by fossil fuel-burning power plants, which pollute the air and contribute to global climate change by emitting greenhouse gases such as carbon dioxide, methane, and nitrous oxide.

Solar technologies can also be a sound economical choice as they can reduce or eliminate exposure to rising electricity rates, or even eliminate one’s need to pay an electrical bill! In addition, solar panels can be a smart long-term investment, with many solar vendors offering 20-30 year warranties on their products.

How does it work?
Solar panels are the most common method for turning sunlight into electricity. Each panel is made up of photovoltaic (PV) cells that are connected electrically and packaged into a frame. These cells, which are constructed of a semiconductor material such as silicon, cause photons from sunlight to knock an electron loose from the semiconductor material. Before this process, the material has been chemically altered to create an electric field, which forces freed electrons to move in a circuit, thus creating electricity.

Name: _____________________
Date: __________ Period ________

1. What are the environmental costs and benefits of solar energy?

________________________________________________________________________

________________________________________________________________________

2. What are the economic costs and benefits of solar energy?

________________________________________________________________________

________________________________________________________________________

3. How would switching to solar energy affect energy use at your home and school?

________________________________________________________________________

________________________________________________________________________

7. In the United States, solar energy accounts for less than one percent of all energy use.\(^{24}\) Why do you think this is? What could be done to increase solar energy use?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

All About Solar
Answer Key

1. What are the environmental costs and benefits of solar energy?
   *Much of the world is racing through its reserves of fossil fuels. Solar energy is a renewable resource, meaning we have a near infinite supply. By using solar energy, we can reduce our reliance on non-renewable forms of energy. Solar energy is also a cleaner form of energy; it creates far less pollution and far fewer greenhouse gases. Pollution is created in the manufacturing and shipment of the panels, but it is far less than the pollution created from burning fossil fuels.*

2. What are the economic costs and benefits of solar energy?
   *In most areas of the country, electricity rates are rising. Solar panels have a high initial cost, but in the long term they can save money by reducing electric bills. Since solar panels are a relatively new field (middle of last century), there is considerable potential for growth in the solar industry, and more jobs may be available in this field in the future for engineers, scientists, solar installers, etc.*

3. How would switching to solar energy affect energy use at your home and school?
   *In general, switching to solar energy would lower your home's electrical costs and reduce your emissions, thus saving money and improving the environment.*

4. In the United States, solar energy accounts for less than one percent of all energy usage. Why do you think this is? What could be done to increase solar energy use?
   *Answers will vary, but more research and development could be done to increase the efficiency of solar panel technology.*

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Quadratic Graphs and Solar Savings

DIRECTIONS: Complete the questions below by filling in the blanks.

1. A small photovoltaic solar electric panel system has been installed at a new school. The daily energy generated by the system can be modeled by the function $y = 64 + 95x - 7.5x^2$, where $x$ is the month and $y$ is power in kilowatt-hours.
   
   a. Does the shape of the graph open UP or DOWN? ______________________________
   
   b. Identify the vertex of the graph. ________________________________________
   
   c. What are the maximum daily sales? ________________________________________
   
   d. Which month will have the maximum energy output? _________________________
   
   e. In which month will the system produce zero energy output? _________________

2. Does your answer to the previous question make sense in real life? Why or why not? The energy output of a huge 2-megawatt photovoltaic solar farm in Arizona can be modeled with the quadratic function $f(x) = 71332 + 63662 - 4868x^2$. (Note: 1,000 watts = 1 kilowatt and 1,000 kilowatts = 1 megawatt.)
   
   a. What does the 71332 in the function represent in your graph/table? ______________
   
   b. What does the 71332 represent in real life in this problem? _________________
   
   c. Find the vertex of the graph of the function. _________________________________
   
   d. What is the maximum energy output of the solar farm? _________________________
   
   e. In what two months is the energy output at about 250 megawatts per month? ______
   
   f. What is the least amount of energy the system will produce? __________________
   
   g. The electricity demand per month of a community of 250 average American homes is approximately 230 megawatts. Between what months of the year can this large solar farm meet this community’s electricity demand? ____________________________
Quadratic Graphs and Solar Savings
Answer Key

DIRECTIONS: Complete the questions below by filling in the blanks.

1. A small photovoltaic solar electric panel system has been installed at a new school. The daily energy generated by the system can be modeled by the function $y = 64 + 95x - 7.5x^2$ where $x$ is the month and $y$ is power in kilowatt-hours.

   a. Does the shape of the graph open UP or DOWN? 
      **DOWN**
   
   b. Identify the vertex of the graph
      *(6.33, 364.83)*
   
   c. What is the maximum monthly energy output?
      **364.83 kilowatt-hours per month**
   
   d. What month will have the maximum energy output?
      **June**
   
   e. In which month will the system produce zero energy output?
      **The 14th month**
   
   f. Does your answer to the previous question make sense in real life? Why or why not?
      **No, the answer does not make sense in real life because there is no 14th month.**

2. The energy output of a huge 2-megawatt photovoltaic solar farm in Arizona can be modeled with the quadratic function $f(x) = 71332 + 63662x - 4868x^2$. (Note: 1,000 watts = 1 kilowatt and 1,000 kilowatts = 1 megawatt.)

   a. What does the 71332 in the function represent in your graph/table?
      **The initial value.**
   
   b. What does the 71332 represent in real life in this problem?
      **The energy output at the zero month.**
   
   c. Find the vertex of the graph of the function.
      *(6.54, 279,469)*
d. What is the maximum energy output of the solar farm?
   
   279,469 kilowatt-hours

e. In what two months is the energy output at about 250 megawatt-hours per month?

   April and September

f. What is the least amount of energy the system will produce in one month during the year?

   130,126 kilowatt-hours of energy - in January

3. The electricity demand per month of a community of 250 average American homes is approximately 230 megawatts. Between what months of the year can this large solar farm meet this community’s electricity demand?

   Between April and September
Exponential Models: Solar PV System

**DIRECTIONS:** Answer the following questions based on the data below. (See Appendix A and Appendix B for Graphing Calculator Instructions.)

<table>
<thead>
<tr>
<th>Year</th>
<th>Yearly Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>$1,175.35</td>
</tr>
<tr>
<td>2011</td>
<td>$1,245.87</td>
</tr>
<tr>
<td>2012</td>
<td>$1,320.62</td>
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<tr>
<td>2013</td>
<td>$1,399.86</td>
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<td>2014</td>
<td>$1,483.85</td>
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<td>2015</td>
<td>$1,572.88</td>
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<td>2020</td>
<td>$2,104.87</td>
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<td>2021</td>
<td>$2,231.17</td>
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</tbody>
</table>

To the left are the yearly savings on the electricity bill for a homeowner in Los Angeles, CA with a 5.5-kilowatt photovoltaic solar panel system. This system offsets 95 percent of the homeowner’s electricity use. With electricity rates increasing a certain percentage every year, the homeowner’s savings increased as well.

1. Enter the data into the lists in your calculator.
   *Note: Assume year 2000 = 0 in calculator.*

2. Accurately draw the scatter plot of the data in the graph on the left using the WINDOW scale below. (Remember to turn on STAT PLOT.)

3. Determine the exponential regression line that fits this data. Write the equation below. Round to the nearest tenth.

4. What is the r or r² value? After looking at the picture of the graph, do you think the r² value reflects how well the line fits the data?

5. Predict what the homeowner’s savings will be in 2025. (Remember what year x=0 is in your table.)
Exponential Models: Solar PV System
Answer Key

**DIRECTIONS:** Answer the following questions based on the data below. (See Appendix A and Appendix B for Graphing Calculator Instructions.)

<table>
<thead>
<tr>
<th>Year</th>
<th>Yearly Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>$1,175.35</td>
</tr>
<tr>
<td>2011</td>
<td>$1,245.87</td>
</tr>
<tr>
<td>2012</td>
<td>$1,320.62</td>
</tr>
<tr>
<td>2013</td>
<td>$1,399.86</td>
</tr>
<tr>
<td>2014</td>
<td>$1,483.85</td>
</tr>
<tr>
<td>2015</td>
<td>$1,572.88</td>
</tr>
<tr>
<td>2016</td>
<td>$1,667.26</td>
</tr>
<tr>
<td>2017</td>
<td>$1,767.29</td>
</tr>
<tr>
<td>2018</td>
<td>$1,873.33</td>
</tr>
<tr>
<td>2019</td>
<td>$1,985.73</td>
</tr>
<tr>
<td>2020</td>
<td>$2,104.87</td>
</tr>
<tr>
<td>2021</td>
<td>$2,231.17</td>
</tr>
</tbody>
</table>

To the left are the yearly savings on the electricity bill for a homeowner in Los Angeles, CA with a 5.5-kilowatt photovoltaic solar panel system. This system offsets 95 percent of the homeowner’s electricity use. With electricity rates increasing a certain percentage every year, the homeowner’s savings increased as well.

1. Enter the data into the lists in your calculator.  
   *Note: Assume year 2000 = 0 in calculator.*

2. Accurately draw the scatter plot of the data in the graph on the left using the WINDOW scale below. (Remember to turn on STAT PLOT.)

3. Determine the exponential regression line that fits this data. Write the equation below. Round to the nearest tenth.  
   \[ y = 656.3(1.06)^x \]

4. What is the r or r² value? After looking at the picture of the graph, do you think the r² value reflects how well the line fits the data?  
   \[ r^2 = 1. The model is a perfect fit for the data. \]

5. Predict what the homeowner’s savings will be in 2025. (Remember what year \( x = 0 \) is in your table.)  
   \$2,816.80
Best-Fit Models: U.S. Energy Consumption

**DIRECTIONS:** The following questions are derived from historical energy data. Enter the data set into your graphing calculator and then answer the questions. (See Appendix A and Appendix B for Graphing Calculator Instructions.)

1. The table below displays the U.S. solar photovoltaic energy consumption in the United States from 2003 to 2008 (in thousands of gigawatt-hours). *Note: Assume year 2000 = 0.*

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption</strong> (GW-hours)</td>
<td>18.76</td>
<td>19.05</td>
<td>19.34</td>
<td>21.10</td>
<td>23.74</td>
<td>28.43</td>
</tr>
</tbody>
</table>

a. Enter the data into the lists of your calculator.

b. Examine the scatter plot of the data.

c. Use an exponential model to determine the prediction equation that fits this data. Write the equation below.

______________________________________________________________________________

d. What is the coefficient of determination (the R² value)? Do you think this best-fit equation is a good fit for the data?

______________________________________________________________________________

e. Using your equation of best fit, predict the consumption of solar in the United States in 2013.

______________________________________________________________________________

Name: ______________________

Date: _________ Period _______
2. The graph below illustrates the total power of world-wide photovoltaic cells between 1992 and 1998 (megawatts of power). Note: Assume 1990 = 0.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>World PV Market</td>
<td>57.9</td>
<td>60.1</td>
<td>69.4</td>
<td>77.7</td>
<td>88.6</td>
<td>125.8</td>
<td>151.7</td>
</tr>
</tbody>
</table>

a. Based on an exponential best-fit model, predict what the world PV market will be like in 2015.

3. The chart below shows the energy generated by a 2.1-kilowatt solar panel system on Northwest Halifax High School in North Carolina. (Note: January = 1 and energy is in kilowatt-hours.)

<table>
<thead>
<tr>
<th>Month of the Year</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Generated</td>
<td>183</td>
<td>306</td>
<td>330</td>
<td>254</td>
<td>224</td>
<td>123</td>
<td>141</td>
</tr>
</tbody>
</table>

a. According to a quadratic best-fit model, how much energy was produced in August?

b. If the actual amount of energy produced in March was 233 kilowatt-hours, what is the difference between the predicted and actual amount?

4. The table below shows the amount of total renewable energy consumed in the United States between 2003 and 2007 in thousands of gigawatt-hours of energy.

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renewable Energy Total</td>
<td>1802.4</td>
<td>1834.6</td>
<td>1881.5</td>
<td>2025.1</td>
<td>1995.8</td>
</tr>
</tbody>
</table>

a. According to the data above, what is the exponential best-fit equation?

b. Is this a great “fit” to the data points?

c. Which polynomial regression model has the best “fit” to the data?
d. Why do you think each polynomial regression function is able to model the data relatively accurately?


a) Based on your model, make a prediction about what consumption will be in 2015.
Lines of Best Fit: U.S. Energy Consumption
Answer Key

DIRECTIONS: The following questions are derived from historical energy data. Enter the data set into your graphing calculator and then answer the questions. (See Appendix A and Appendix B for Graphing Calculator Instructions.)

1. The table below displays the U.S. solar photovoltaic energy consumption in the United States from 2003 to 2008 (in thousands of gigawatt-hours).

   Note: Assume year 2000 = 0.

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>18.76</td>
<td>19.05</td>
<td>19.34</td>
<td>21.10</td>
<td>23.74</td>
<td>28.43</td>
</tr>
</tbody>
</table>

   a. Enter the data into the lists of your calculator.

   b. Examine the scatter plot of the data.

   c. Use an exponential model to determine the prediction equation that fits this data. Write the equation below.

   Assuming the year 2000 is zero, the equation will be: \( y = 13.78(1.08)^x \)

   d. What is the correlation coefficient (the \( R^2 \) value)? Do you think this best-fit equation is a good fit for the data?

   The correlation coefficient value is 0.86. It is a good, but not a great, fit.

   e. Using your equation of best-fit, predict the consumption of solar in the United States in 2013.

   According to the model, the United States will consume 3,937,000 gigawatt-hours of solar energy in 2013.

2. The graph below illustrates the total megawatts of power produced by photovoltaic cells worldwide between 1992 and 1998. Note: Assume 1990 = 0.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>World PV Market (MW)</td>
<td>57.9</td>
<td>60.1</td>
<td>69.4</td>
<td>77.7</td>
<td>88.6</td>
<td>125.8</td>
<td>151.7</td>
</tr>
</tbody>
</table>

   a) Based on an exponential best-fit model, predict what the world PV market will be like in 2015.

   Based on the model, the world PV market will produce 2286.7 megawatts of power in 2015.
3. The chart below shows the energy generated by a 2.1-kilowatt solar panel system on Northwest Halifax High School in North Carolina. (Note: January = 1 and energy is in kilowatt-hours.)

<table>
<thead>
<tr>
<th>Month of the Year</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Generated</td>
<td>183</td>
<td>306</td>
<td>330</td>
<td>254</td>
<td>224</td>
<td>123</td>
<td>141</td>
</tr>
</tbody>
</table>

a. According to a quadratic best-fit model, how much energy was produced in August?
   According to the quadratic model of \( y = 61.3 + 79.5x - 6.25x^2 \) and assuming August is the eighth month, the system will produce about 297 kilowatt-hours of energy in August.

b. If the actual amount of watt-hours of energy produced in March was 233 kilowatt-hours, what is the difference between the predicted and actual amount?
   The difference between the model and the actual amount is about 10.6 kW.

4. The table below shows the amount of total renewable energy consumed in the United States between 2003 and 2007 in thousands of gigawatt-hours of energy.

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renewable Energy Total (GWh)</td>
<td>1802.4</td>
<td>1834.6</td>
<td>1881.5</td>
<td>2025.1</td>
<td>1995.8</td>
</tr>
</tbody>
</table>

a. According to the data above, what is the exponential best-fit equation?
   The equation is \( y = 1638(1.03)^x \)

b. Is this a great “fit” to the data points?
   No, the \( r^2 \) value is only 0.87.

c. Which polynomial regression model has the best “fit” to the data?
   A quadratic regression.

d. Why do you think each polynomial regression function is able to model the data relatively accurately?
   Because of the limited number of data points.

e. Based on your model, predict what energy consumption will be in 2015.
   Based on the model, energy consumption will be 2,579,500 gigawatt-hours.
Solar Power and Me: The Financial Savings

**DIRECTIONS:** The following questions are derived from historical energy data for the average America residential single-family home. Answer these questions in complete sentences and show your work.

1. Make a guess: What uses more energy in homes — heating, cooling, lighting or powering electronics like TVs, computers and MP3 players? Do houses in the United States use more electricity or more natural gas? Answer in at least two complete sentences.

**Background Information:** In 2010, the average single-family home in the U.S. consumed 920 kilowatt-hours of electricity per month. The average retail price was 11.26 cents per kilowatt-hour. Thus, the average monthly bill for a single-family home in 2010 was $103.67 per month.

2. Imagine you installed a fairly standard 2-kilowatt (kW) photovoltaic solar panel system on your home that produces an average of 300 kWh of energy per month.
   a. If the price of electricity is 11.26 cents per kWh of energy, how much will this system save you per month in electricity costs?
   b. If this system costs you $15,000 and your home’s average energy consumption remains at 300 kWh per month, how long will it take to break even (that is, for your savings to pay for the cost of the system)?
   c. If a 6 kW system produced an average of 900 kWh of energy per month and cost $50,000 to install, how long would it take you to break even?

3. Now imagine you live in amazing and sunny Anaheim, CA, where the combination of local and federal rebates covers 74 percent of your total cost of a solar panel system.
   a. You now pay $112.03 per month on your solar electric bill. The cost of electricity is 12.2 cents and your new 5.5-kW solar panel system will generate 745 kWh per month. How much will you save per month? What will your new electric bill be?
b. Assuming these savings continue for 20 years, how much will you save in that time?

_____________________________________________________________________

4. The electricity rate is $0.1224 per kilowatt-hour and it increases at 6 percent per year.

a. What exponential formula could model the growth in price of electricity for any given year in the future?

_____________________________________________________________________

b. What will be the cost of electricity 20 years from now?

_____________________________________________________________________

c. How much will you save over 20 years?

_____________________________________________________________________

5. The estimated cost of this solar panel system is $31,738.

a. How much would you have to pay after receiving local and federal incentives, which cover 74 percent of the costs?

_____________________________________________________________________

b. If you took out a loan to cover the costs of your new system after rebates, at an annual interest rate of 8 percent and a length of 20 years, your monthly payment on the loan will be $66.92. How much will you save on your electric bill each month in the first year? In the last year?

_____________________________________________________________________
Solar Power and Me: The Financial Savings

Answer Key

DIRECTIONS: The following questions are derived from historical energy data for the average America residential single-family home. Answer these questions in complete sentences and show your work.

1. Make a guess: What uses more energy in our homes—heating, cooling, lighting or powering electronics like TVs, computers and MP3 players? And, do houses in the United States use more electricity or natural gas? Answer in at least two complete sentences.
   
   Space heating in U.S. homes uses the most energy. Lighting and powering small electronics comes in second place. Houses in the United States use slightly more natural gas than electricity.

2. Imagine you installed a fairly standard small 2 kilowatt (kW) photovoltaic solar panel system on your home that produces an average of 300 kWh of energy per month.
   
   a. If the price of electricity is 11.26 cents per kWh of energy, how much will this system save you per month in electricity costs? $33.78

   b. If this system costs you $15,000, and at this same average rate of 300kWh of energy per month, how long will it take you to break-even on the systems (i.e., when your savings pay for the cost of the system). 37 years

Background Information: In 2010, the average electricity consumption was 920 kilowatt-hours of energy per month. The average retail price was 11.26 cents per kilowatt-hour. Thus, the average monthly bill for a single-family home in 2010 was $103.67 per month.

Types of Energy Consumed in Homes, 2005

<table>
<thead>
<tr>
<th>Energy Type</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Gas</td>
<td>45%</td>
</tr>
<tr>
<td>Electricity</td>
<td>41%</td>
</tr>
<tr>
<td>Fuel Oil</td>
<td>8%</td>
</tr>
<tr>
<td>Propane</td>
<td>5%</td>
</tr>
</tbody>
</table>

How Energy Is Used in Homes (2005)*

- Space Heating: 41%
- Lighting and Other Appliances: 26%
- Water Heating: 20%
- Air Conditioning: 8%
- Refrigeration: 5%

* 2005 is the most recent year for which data are available.

c. If a 6 kW system produced an average of 900 kWh of energy per month and cost $50,000 to install, how long would it take you to recoup your money? 
   **41 years and 1.3 months**

3. Now imagine you live in amazing and sunny Anaheim, CA where the combination of local and federal rebates covers 74 percent of your total cost of a solar panel system!

   a. You previously paid $112.03 per month on your solar electric bill. The cost of electricity is 12.2 cents and your new 5.5 kW solar panel system will generate 745 kWh per month. How much will you save per month? What will your new electric bill be?  
      **90.89 and you will save $21.14 per month.**
   
   b. Assuming electricity rates do not increase and efficiency stays the same and this savings continues for 20 years, how much will you save?  
      **$5,073.60**

4. The electricity rate is $.1224 per kilowatt-hour and it increases at 6 percent per year.

   a. What exponential formula could model the growth in price of electricity for any given year in the future?  
      \[ f(x) = 0.1224(1.06)^x \]
   
   b. What will be the cost of electricity 20 years from now?  
      **$0.39**
   
   c. How much will you save over 20 years?  
      **$290.55**

5. The estimated cost of this solar panel system is $31,738.

   a. How much would you have to pay after local and federal incentives covered 74 percent of the costs?  
      **$8251.88**
   
   b. If you took out a loan to cover the costs of your new system (after rebates), at an annual interest rate of 8 percent and a length of 20 years, your monthly payment on the loan will be $66.92. How much will you STILL save on your electric bill each month in the first year? In the last year?  
      **$23.97 and $223.63**
Exit Ticket – Energy for Life

Given the following data on an actual solar photovoltaic system on the roof of a school in North Carolina, create a best-fit quadratic regression model to accurately reflect the data. Note: Time is in 24-hour periods, so time 16 = 4:00 p.m.

<table>
<thead>
<tr>
<th>Time</th>
<th>Watts</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>52</td>
</tr>
<tr>
<td>8</td>
<td>445</td>
</tr>
<tr>
<td>10</td>
<td>1030</td>
</tr>
<tr>
<td>11</td>
<td>1470</td>
</tr>
<tr>
<td>12</td>
<td>1170</td>
</tr>
<tr>
<td>15</td>
<td>1470</td>
</tr>
<tr>
<td>16</td>
<td>981</td>
</tr>
<tr>
<td>18</td>
<td>410</td>
</tr>
<tr>
<td>19</td>
<td>89</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

1. Make a prediction for what the watts produced will be tomorrow at 2 p.m.). Assume the weather will be the same.

2. If the system is producing 1000 watts, what two times could it be?

3. How accurate is your model?

4. What factors do you think influence how much solar power a particular system generates each day?

5. What type of shape do you think the graph produces?
Exit Ticket – Energy for Life
Answer Key

Given the following data on an actual solar photovoltaic system on the roof of a school in North Carolina, create a best-fit quadratic regression model to accurately reflect the data. Note: time is in 24-hour time, so time 16 = 4:00 p.m.

<table>
<thead>
<tr>
<th>Time</th>
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<tbody>
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<tr>
<td>8</td>
<td>445</td>
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<tr>
<td>10</td>
<td>1030</td>
</tr>
<tr>
<td>11</td>
<td>1470</td>
</tr>
<tr>
<td>12</td>
<td>1170</td>
</tr>
<tr>
<td>15</td>
<td>1470</td>
</tr>
<tr>
<td>16</td>
<td>981</td>
</tr>
<tr>
<td>18</td>
<td>410</td>
</tr>
<tr>
<td>19</td>
<td>89</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

1. Make a prediction for what the watts produced will be tomorrow at (2:00 pm). Assume the weather will be the same.
   **1314 watts**

2. If the system is producing 1000 watts, what two times could it be?
   9:42 a.m. (9.7) and 4:18 p.m. (16.3)

3. How accurate is your model?
   *The model has an r coefficient of 0.91 so it is a strong representation of the data.*

4. What factors do you think can influence how much solar power is generated each day on a particular system?
   *The location, orientation, efficiency, and weather (cloud cover) all determine how much power a solar system generates.*

5. What type of shape do you think the graph produces?
   *The graph is in the shape of a parabola.*
Appendix A
Calculator Regression Model Guide for Students

A mathematical model is an equation that best describes a particular set of paired data. Such models are referred to as regression models and commonly known as “lines of best fit.” They are used to make predictions about one variable based on another variable.

The Basics

I. Clearing Data:

1. To clear all data from a list, press STAT. From the EDIT menu, move the cursor up onto the name of the list (L1). Press CLEAR. Move the cursor down.

2. The list entries will not disappear until the cursor is moved down. (Avoid pressing DEL as it will delete the entire column. If this happens, you can reinstate the column by pressing STAT #5 SetUpEditor.)

3. To clear an individual entry, select the value and press DEL.

II. Entering Data:

Consider the data set from Reproducible #3 – Best-Fit Models: Solar Savings

1. Data is stored in Lists on the calculator. Locate and press the STAT button on the calculator. Choose EDIT. The calculator will display the first three of six lists (columns) for entering data. Simply type your data and press ENTER. Use your arrow keys to move between lists.

2. Enter the X data values in L1. Enter the Y data values in L2, being careful to enter each X data value and its matching Y data value on the same horizontal line.

III. Scatter Plot:

1. Activate the scatter plot. Press 2nd STATPLOT and choose #1 PLOT 1. You will see the screen at the right. Be sure the plot is ON, the scatter plot icon is highlighted, that the list of the X data values are next to X list, and the list of the Y data values are next to Y list. Choose any of the three marks.

2. To see the scatter plot, press ZOOM and #9 ZOOMSTAT.
Correlation

How well does your regression equation truly represent your set of data?

One of the ways to determine the answer to this question is to examine the correlation coefficient and the coefficient of determination.

The correlation coefficient, \( r \), and the coefficient of determination, \( r^2 \), will appear on the screen that shows the regression equation information. (Be sure the Diagnostics are turned on – 2\textsuperscript{nd} Catalog (above 0), arrow down to DiagnosticOn, press ENTER twice.)

**Correlation Coefficient, \( r \):**

The quantity \( r \), called the linear correlation coefficient, measures the strength and direction of a linear relationship between two variables.

The value of \( r \) is such that \(-1 \leq r \leq +1\). The + and – signs are used for positive linear correlations and negative linear correlations, respectively.

- **Positive correlation:** If \( x \) and \( y \) have a strong positive linear correlation, \( r \) is close to +1. An \( r \) value of exactly +1 indicates a perfect positive fit. Positive values indicate a relationship between \( x \) and \( y \) variables: as values for \( x \) increase, values for \( y \) also increase.

- **Negative correlation:** If \( x \) and \( y \) have a strong negative linear correlation, \( r \) is close to -1. An \( r \) value of exactly -1 indicates a perfect negative fit. Negative values indicate a relationship between \( x \) and \( y \): as values for \( x \) increase, value for \( y \) decrease.

- **No correlation:** If there is no correlation or a weak correlation, \( r \) is close to 0. A value near 0 means that there is a random, nonlinear relationship between the two variables. Note that \( r \) is a dimensionless quantity. Specifically, it does not depend on the units employed.

- **Perfect correlation:** A correlation of \( \pm 1 \) occurs only when the data points all lie exactly on a straight line. If \( r = +1 \), the slope of this line is positive. If \( r = -1 \), the slope of this line is negative. For example, a correlation greater than 0.8 is generally described as strong, whereas a correlation less than 0.5 is generally described as weak. These values can vary based on the type of data being examined. A study using scientific data may require a stronger correlation than a study using social science data.
**Coefficient of Determination or $r^2$:**

The **coefficient of determination or $r^2$** is useful because it gives the proportion of the variance (fluctuation) of one variable that is predictable from the other variable. This measure allows us to determine how certain we can be in making predictions from a certain model/graph.

The **coefficient of determination** is the ratio of the explained variation to the total variation, such that $0 \leq r^2 \leq 1$. It denotes the strength of the linear association between $x$ and $y$.

The **coefficient of determination** represents the percentage of the data closest to the line of best fit. For example, if $r = 0.922$, then $r^2 = 0.850$, which means that 85 percent of the total variation in $y$ can be explained by the linear relationship between $x$ and $y$ (as described by the regression equation). The other 15 percent of the total variation in $y$ remains unexplained.

The **coefficient of determination** is a measure of how well the regression line represents the data. If the regression line passes exactly through every point on the scatter plot, it would be able to explain all the variation. The further the line is away from the points, the less it can explain.

**Turn on Diagnostics!**

The Diagnostic flag must be on for you to see your $r$ and $r^2$ values.

*Note: When your calculator is reset (or the default is set), your Diagnostic flag will be turned off. You will need to turn your Diagnostics back on.*

**To turn Diagnostics on:**

1. Press 2nd CATALOG (above the numeral zero) to display the catalog in alpha mode. (Note the A in the upper right hand corner.)

2. Press D (to fast forward to the Ds) and use the down arrow ▼ to move the pointer to DiagnosticOn.

Appendix C
Guided Calculator Notes for Teachers with Answer Key

Step 1. Enter the data into the lists.
Press STAT, then ENTER. Enter the x-values into L1 and the y-values into L2.

Step 2. Create a scatter plot of the data.
Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON and set the icon to Scatter Plot (the first one), set Xlist to L1, and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice.

Step 3. Choose Exponential Model.
Press STAT, arrow right to CALC, and arrow down to 4 ExpReg. Hit ENTER. When ExpReg appears on the home screen, press VARS, then arrow over to Y-VARS and press ENTER twice. This will put the equation into Y= for you.

Step 4. Graph the Exponential Regression Equation from Y1.
Press ZOOM #9 ZoomStat to see the graph.
Step 5. Is this model a "good fit"?
The correlation coefficient, $r$, is exactly 1, which places the correlation into the "perfect" category (0.8 or greater is a strong correlation).

Step 8. Make a prediction (extrapolate data).

*In 2025, how much will the homeowner’s annual electricity savings be?*

**Method 1:** With your exponential equation in Y1, go to the home screen and type Y1(25). Press ENTER

**Method 2:** Press 2ND TBLSET and enter 25 for TblStart. Then go to your table

Both methods reveal the same answer: in 2025, the annual savings would be $2,816.80. (answer to question 5)