Plato's Premise: Fostering Student Autonomy

by Robert Case

Billie Holiday sings, and the sorrow and the joy, the loss and the gain of life flows through her art to us as listeners. Edward Albee's play unfolds, and the emotions and ideas ripple through the theater as it becomes a crucible of action and passion. James Joyce's meticulous attention to human detail transfuses into art in the character of Leopold Bloom, and this portrayal of the human heart awes the reader. Ingmar Bergman directs, and in dialogue and action the film holds up a mirror to our own experience.

The teacher, aiming to nurture a love of learning in students, asks: Is it possible to teach with a piece of Billie Holiday's heart and soul? With a bit of Albee's capacity to transform our understanding? With an eye on Joyce's insight? With Bergman's power to be an architect of personal reflection?

Like the arts of literature, drama, film, and music, teaching has an identity that grows from roots stretching back to antiquity. Good teaching causes the seeds of Greek fundamentals to blossom into the evocative and improvisational qualities of the modern arts, leaving space for the student to become fully active, to learn and grow. In concrete terms, what beliefs

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does the teacher bring to his or her craft? What does he or she try to elicit from the student? We need to know because these two actions—teacher reflection and student activity—are the two parts that become fused in the art of conscious teaching.

The classroom, like Edward Albee's theater, becomes the forum where we can sense that something unique, change-producing, and personal is about to happen—where on a good day something bright, even lasting is produced.

Thomas Aquinas famously observed that 'whatever is received is received according to the qualities of the receiver,' not the giver.

Decades ago many of us came into the classroom to lecture to a basically unknown group of young people whose names appeared on a list generated by the registrar. All too often, the students continued to be strangers throughout the course. We now know that this was not the best approach; we now know we must encounter these students as persons, continually communicating to them three things:

- You, the student, can appreciate mathematics as a beautiful subject.
- I, the teacher, really want you to learn this subject.
- You, the student, are capable of learning and using this subject, with a measure of real creativity and autonomy.

These beliefs in the capability of our students are central because they determine the context, the culture, and the atmosphere of every part of our courses. They govern the level of discourse in our teaching, because we "speak" differently to people about whom we have different expectations. The powerful constructive or destructive consequences of our cultural assumptions have been studied at the precollege level, and college experience tells us of their importance in higher education as well.[1]

Thomas Aquinas famously observed that "whatever is received is received according to the qualities of the receiver," not the giver. We must keep this in the very forefront of our teaching, because ignoring the social and emotional dimensions of our work results in ineffective teaching and, eventually, the weakening of our disciplines.

In my experience, the core of the interaction between teacher and student operates within two related activities: Socratic dialogue and problem solving. The teacher's reflection and the student's action both become integral to the art of teaching as they meet in the forum where dialogue and problem solving take place. The overall goal of this art is students' intellectual autonomy.

Socratic dialogue is built on teachers' carefully constructed sequences of questions to elicit in students the discovery of an idea or method. As mathematician George Polya puts it, the instructor tries "to help the student naturally. The teacher should put himself in the student's place…he should try to understand what is going on in the student's mind, and ask a question that could have occurred to the student himself."[2]

In my discipline, this means that the flow of the mathematics should come from the student rather than the instructor. In the 1980s, a colleague and I received a university grant to teach a carefully monitored course in calculus. Our task was to break down the abstract narrative of the math text into sequences of questions—even several branches of such sequences—in anticipation of various kinds of student responses. Socratic dialogue brings the classroom into the rhythm of the way a mathematician actually thinks.

There is the silence at the heart of dialogue: the "wait-time" between the questions and the response, and the silence between a question and a new question. These are the silences of growth. This came home to me when I asked a student what he wanted in a teacher. His response: "I need a teacher who really listens to me, and then builds on my idea." This kind of teaching involves the use of productive silence.

There are all kinds of questions the teacher can ask. Specific ones: "How can you find the area of this rectangle?" and ones that are open-ended: "What can happen as the number of these rectangles increases beyond any fixed number, however large?" The questions can be procedural: "How many agree with Jose?" or general: "How is this problem different from the previous one?" The questions can be fashioned so as to speed up or to slow down the material; so as to focus on the concepts in

some cases, and the notation in others.

These methods are, in a fashion, art. They are analogous to Bergman's directorial strategies—the use of the cross-cut, the angle shot, the fruitful pause, the bird's eye view. They elicit reflective involvement in the student, who becomes a participant in the creative process. As instructor and students become more accustomed to this process, there is even a high probability of getting a constructive response from asking "What is the next question?"

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Using Socratic Dialogue causes students to become questioners themselves. When I began teaching an entire course through questioning, it turned out to have a great effect on students: They became very enthusiastic and active. One student who began the course aggressively became cooperative in the give-and-take of dialogue and another, who started out shyly, developed an outgoing personality in class meetings. These students did significantly better in the standardized final exam than other groups not taught by Socratic dialogue, and they evaluated the course very highly, wanting to know how they could continue to study math "this way."

As the instructor, I was evaluated less highly than the course. This was a terrific outcome since the students expressed the sense that they were teaching themselves the math. I had become virtually invisible, a coach, not a leader; an architect, not a builder; a director, not an actor. The students were beginning to become autonomous learners.

Socratic dialogue strives for creating "Aha!" moments of discovery in learners. Take for example the case of the geometric series. Students might confront a fruitful problem such as the following:

"Suppose there is a dump in your town that opens up with a delivery of 2,000 tons of refuse on the first day of the month, and then 2,000 tons on the first day of each month thereafter. Because of composting, only 80 percent of what was in the dump at the start of previous month remains when the first-of-the-month delivery of the new 2,000 tons is made. So on the first day of the second month you get 2,000 (new delivery) + 2,000 x 0.8 (what remains) for a total of 3,600 tons. (This is a geometric series.)

Question: "Will the dump run out of space as time extends indefinitely, or will a fixed size be enough to accommodate everything that eventually gets delivered?"

Each group of students to whom I've posed this problem has discovered a compelling reason for answering that a dump of fixed size will do. Usually the explanation follows this line: If the amount in the dump gets really big over time, the 20 percent it loses during the most recent month will also be very big and will easily accommodate the incoming 2,000 tons. Therefore, the dump's accumulation can't really grow beyond a certain point. Once the students arrive at this basic insight, they are ready to go on and do a lot of other mathematics connected to geometric series.

As the art of teaching has come to focus on the student as the active center of learning, it's surprising how the strategies and emphases associated with the principal Greek philosophers reassert themselves. Besides Socrates (whose name is associated with questioning dialogue), there are Euclid (formal exposition), Aristotle (experiments and applications) and Plato (nurturing the autonomous learner).

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Although the art of teaching described here centers on dialogue and problem solving and links to Socrates, which in turn leads to the autonomy central to Plato's conception of society, nevertheless a portion of this art is shaped by the careful exposition of Euclid. It is also influenced by emphasizing the experience that comes from Aristotelian contact with objects.

Those of us who attended school several decades ago looked forward to polished lectures. The Euclidean rigor and the logical structure of the exposition were certainly powerful and beautiful. But this is not the way most people learn mathematics, and trying to bring the style of the lectures received in graduate school to one's own teaching leads to a severe jolt.

This doesn't mean we should abandon rigor, but rather where we go as teachers depends on the situation of the students and the purpose of the course. This leads to transforming the exposition of Euclid into the inquiry of Socrates.

The movement of Greek thought to an emphasis on direct contact with objects and experimentation is often attributed to Aristotle, and contemporary teaching parallels this dimension by including student projects growing out of applications in the course of study. Students express phenomena such as the growth of crystals, the heating of a building, or the rate of pollution cleanup in mathematical models. Polished explanations, often produced with the help of graphing calculators, computers, or other technology, extend the student-generated mathematics begun in Socratic dialogue. With ingenuity and reflective teaching, technology provides the tools to deal with real data, and to explore and make conjectures in ways that would have been unthinkable even 20 years ago.

Intellectual autonomy, usually associated with Plato, is the overarching goal of teaching. Teachers develop students' autonomy in mathematics through dialogue but also through problem solving, often group problem solving.[3] As an example of group problem solving, I conducted a seminar with a dozen secondary school teachers. We had the chance to study the revolution in thought brought about almost singlehandedly at the end of the 19th century by mathematician Georg Cantor, who found a way to handle the infinite. The participants discussed and explored this revolution in thought, which has provided much of the structure of modern mathematics. Step by step, they delved into one powerful example, Cantor's Middle Thirds Set. Small groups carried out a series of fruitful mini-problems about this Cantor Set, the participants reported on and discussed their findings, and, eventually, they found solutions to these and other questions.

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Some of these questions cascaded into the dynamic "recent" topics of fractals and chaos. Fractal entities and chaotic behavior simulated over and over in natural and technical phenomena, such as the shape of coastlines or the transmission of information.

The sense of community among the learners deepened as the group grew more eager, asking and answering questions and problems as they occurred. There was a corresponding growth in confidence, participants grew as autonomous intellectuals. It is a measure of the power of group problem solving that different groups often came up with widely varying but equally valid arguments for the same conclusion.

To me, the fundamental goal of teaching is the nurturing of autonomy, a term that here includes intellectual initiative and creativity—the capacity to frame questions and problems, and to develop the means to solve these problems. Among the Greek thinkers who stand as beacons for the themes that are embroidered into effective teaching, Plato is the spokesperson for the goal of autonomy.

Group or individual problem solving extends Socratic dialogue and takes student autonomy to an even richer level, since the answers are not given and the students must fully justify their positions. And here the role of silence and patience in the teacher is even more significant than in Socratic dialogue alone. Though the curriculum is carefully shaped by the instructor, the authority over the subject passes subtly to the student.

The parallel is strong with Plato's political thought. Every citizen of the Republic was to be given the opportunity to develop their potential to the utmost, to have the fullest access to education so that eventually the wise leaders of society could emerge from the entire population.[4] This society would no longer be based on heredity but on equal access. Education would lead to genuine autonomy and independent thought.

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The challenge of the teacher in the 21st century is to construct a modern setting rooted in these themes. James Joyce's literary innovations are built on a thorough grasp of classical literature—John Coltrane's improvisational genius grows from a comprehensive musical sensibility; and Ingmar Bergman's creative silence arises from distilling the most traditional philosophical questions. In each of these cases, the reader—hearer, viewer—is drawn into a creative response because a space has been made for him or her by the evocative qualities that are the hallmarks of the modernity of contemporary literature, music, and cinema.

The teacher also tries to transmute traditional themes into contemporary forms of dialogue and problem solving with the result that the student is unable to be a mere observer, but must become a participant. Participation leads to autonomy.

We are most aware of being teachers in that moment when a student experiences the spark of discovery. But we also savor being teachers when we meet the engineering student, now an engineer, years later in a supermarket parking lot, or hear from a recent graduate who worked with us as a tutor and decided out of the blue to become a teacher. They have all grown more autonomous, in their own knowledge and application of math and in their personal self-confidence and creativity.

My guess is that if I did not have this goal of autonomy at the heart of my teaching, my efforts would often be rudderless and disconnected. And if students do not move toward creativity and self-possession and the responsible use of their education, there is an unfortunate alternative that opens up—a modern world that is little more than "a wilderness of mirrors".[5] In contrast, students asked to write down reasons for studying mathematics suggested: "It helps you understand life," "It is a very interesting and beautiful subject," or "By understanding math, you will understand the universe and the things that surround you…it is like an open door to understanding."

Raphael's painting, *The School of Athens,* places all the Greek pioneers of thought in one scene. In the center are Plato and Aristotle, and on the right, Euclid—the geometer—is among the painters. On the opposite side, within a group, is the seeker, Socrates. It is significant that these lovers of learning, including many from different periods of Greek history, have been gathered by Raphael into one comprehensive portrait. Similarly, I've come to think of teaching as the art weaving the pedagogical characteristics associated with these thinkers into a single seamless process. In this process, the expository genius of Euclid is reshaped into the central vehicles of Socratic dialogue and problem solving, enhanced by Aristotle's emphasis on experiment. And these are incorporated in a contemporary setting in which improvisation and inquiry elicit a deep student involvement. In all this, the fundamental goal is to nourish—in the art produced by the marriage of the teacher's reflection and the student's rich activity—the intellectual autonomy of the Plato's wise citizen and leader. nea

ENDNOTES

[1] Oakes, 1992, 12-21; Ladson-Billings, 1997, 697-708; Atweh, Michael, et al., 1998, 63-82.

[2] Polya, 1957, 1.

[3] Davidson, 1990, 335-361.

[4] Hamilton, 1961.

[5] Paz, 1961, 21.

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